**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Solution:**

We have a normal distribution with = 45 and = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50 so the question is to find

Pr(X > 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - )/

= (X - 45)/8.0

Thus, the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0)

= Pr(Z ≤ 0.625)

=73.4%

Probability that the service manager will not meet his demand will be

= 100 - 73.4 = 26.6% or 0.2676

**Answer is B. 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Solution:**

We have a normal distribution with = 38 and = 6.

Let X be the number of employees.

So according to question

1. Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X ≤ 44).

Z = (X - 38)/6

Thus the question can be answered by using the normal table to find Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6)

= Pr(Z ≤ 1)

=84.1345%

Probability that the employee will be greater than age of 44 = 100 84.1345=15.86%

So, the probability of number of employees between 38 - 44 years of age = Pr(X<44)-0.5

=84.1345-0.5

= 34.1345%

**Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.**

1. Probability of employees less than age of 30 = Pr(X<30).

Z = (30 - 38)/6

Thus, the question can be answered by using the normal table to find

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6)

= Pr(Z ≤ -1.333)

=9.12%

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400 = 36.48(or 36 employees).

**Therefore, the statement B of the question is also TRUE.**

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Solution:**

As we know that if

X ∼ N (µ1, σ1^2), and Y ∼ N (µ2, σ2^2)

are two independent random variables then

X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2), and

X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY,

where X and Y are as defined above,

i.e., Z is linear combination of X and Y, then

Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2X1~ N (2 u,4 σ^2) and

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Solution:**

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The Probability of getting value between a and b should be 0.99.

So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (i.e., 1-0.99).

The Probability towards left from a = -0.005 (i.e., 0.01/2).

The Probability towards right from b = +0.005 (i.e., 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z = (X- μ) / σ For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z (-0.005) \* 20 + 100 = -(-2.57) \* 20 + 100

= 151.4

Z (+0.005) \* 20 + 100 = (-2.57) \* 20 + 100

= 48.6 So,

**Answer:** Option D.48.5, 151.5 is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Solution:**

Given that:

$1 = Rs. 45

*Profit1 ~ N (5,32)*

*Profit2 ~ N (7,42)*

Thus, Company's profit:

P ~ N (5 + 7, 32 + 42) = N (12, 52)

1. 95% of the probability lies between 1.96 standard deviations of the mean. Thus,

Range = (12 – 1.96\*5, 12 + 1.96\*5)

= ($2.2M, $22.8M)

= (Rs.99M, Rs.1026M)

1. Fifth percentile is calculated as:

From p values of z score table, we get:

= -1.644

P = 12 – 8.22

= 3.78

Thus, at $3.78M dollars, or Rs. 170.1M amount, 5th percentile of profit lies. Or 5th percentile of profit is Rs. 170.1 million.

1. Loss is when profit < 0

Thus, p < 0

The first division of company, thus have larger probability of making a loss in a given year